Black Hole Geometry

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Introduction

This booklet is for the black hole spacetime models that are part of *Crochet The Universe*: a series of patterns for various space-y physics concepts for you to crochet. The idea behind the patterns is for higher level physics concepts such as black holes to be made into easily visualisable and holdable models that can be used for classroom demonstrations, or they can be made just for fun!

Yarn & Hooks

The models will be small enough to be held with one hand, however you are welcome to use any weight of yarn and any hook that makes your stitches nice and tight, as **there is no need to gauge swatch**. Expect a larger model when using aran and chunky yarn, and a smaller one when using sport/fingering etc.

The model in this book used dk weight acrylic yarn **(Stylecraft Special dk)** with a 4 mm (US G) hook. Once again, use whatever size and hook you please, this isn't to-scale anyways!

Almost all types of yarn are suitable for this. Stylecraft special dk was chosen as it is usually the most affordable option, but I highly recommend going through your stash for small balls from previous projects.

For the models, I used the following colours:
1003 - Aster
1023- Raspberry
1001 - White (for stitching the lines)

Abbreviations

All patterns are in **US terms** and with standard crochet abbreviations. A list will be provided below as a reminder:

sc single crochet
dc double crochet
hdc half-double crochet
sl-st slip-stitch
ch chain

hdc2tog half-double crochet 2 together
sc2tog single crochet 2 together

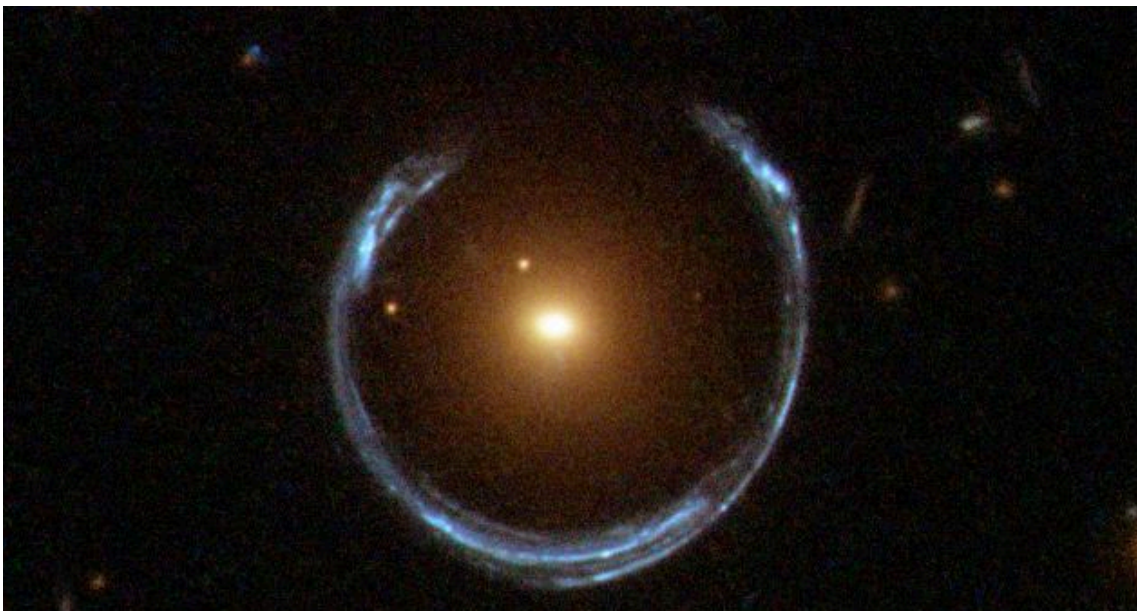
Black Hole Spacetime Geometry

Note: Solar Mass = the mass of the Sun (2 x 10^30 kg). 2 solar masses means twice as heavy as the Sun.

What is a Black Hole?

Black holes are the final stage of large stars in the current universe. These large stars (usually over 8 Solar masses) will stop being able to fuel themselves with nuclear fusion, and their gravity will cause them to collapse to the point where not even electrons and neutrons can hold up the star. This is why stellar sized black holes are sometimes called collapsars.

There are also supermassive black holes (SMBH) that exist in almost all large galaxies, but their origin is still unknown. These kinds of black holes are thousands to billions of solar masses, and ones that are accreting matter and producing high luminosities are called active galactic nuclei (AGN). No scientist yet knows how supermassive black holes came to be, and if they are just stellar black holes that have grown, how those stellar black holes managed to grow so quickly.



This orange fuzzy galaxy is so dense and has such a big black hole that it's bending the galaxy behind it's light into a ring! This is called an Einstein ring. Credit: ESA/Hubble.

Black holes are so dense and have such high mass & gravity that spacetime warps around them and distorts light. This is really useful for measuring mass, because more mass means more distortion. Black holes bend space close to them into a specific shape, called a Flamm's paraboloid, and those that rotate have an extra component called frame-dragging.

Types of Black Hole

Because light cannot escape a black hole once it passes the event horizon, we can only see 3 properties: mass (we can see that from the distortion), electric charge (not-so-easy to tell), and angular momentum (spinning). Every black hole needs mass, but they might have a combination or none of the other two properties as well.

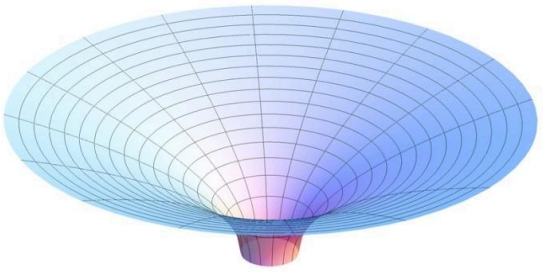
This means there are 4 types of black hole: Schwarzschild (mass only), Reissner-Nordström (mass + charge), Kerr (mass + rotation), and Kerr-Newman (all three).

What's interesting is that charges around a black hole like to neutralize, negative charges attract positive and balance. So in general, black hole can be considered neutral/chargeless, and for the purpose of the crochet models we're about to make, we don't need to consider Reissner-Nordström and Kerr-Newman physics!

Think of Schwarzschild black holes as plain artificial vanilla ice cream, and Kerr black holes as proper Madagasan vanilla. Kerr-Newman could perhaps be Madagascan vanilla with sprinkles and sauce: nice but not necessary to enjoy the taste of the ice cream!

Schwarzschild black holes bend spacetime around them into a Flamm's paraboloid, which is described by the formula below and looks like this:

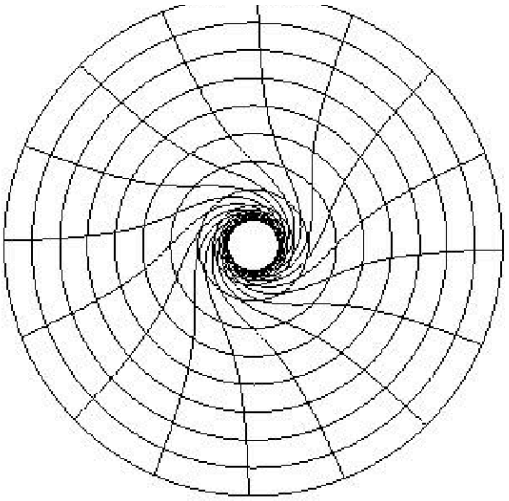
$$w = 2\sqrt{r_s\,(r - r_s)}.$$



Credit: Allen MC

What's interesting is that the paraboloid doesn't start from one tiny point like regular shapes, but it starts with a small ring at the bottom. The explanation for this is in the formula above. The start of the paraboloid is when w = 0, and to get that you need r to equal the r_s term. This is the radius of the black hole(the event horizon), and that makes sense because this spacetime distortion must start right outside the black hole.

When the black hole rotates, however, the paraboloid distorts a bit more! The curvature of spacetime is somewhat distorted with the rotation of the black hole, as though it were spinning with the black hole! This is called frame dragging, and that alone is extremely difficult to visualise. P.C Van der Wijk in their thesis graphs out a visualisation for this frame dragging:



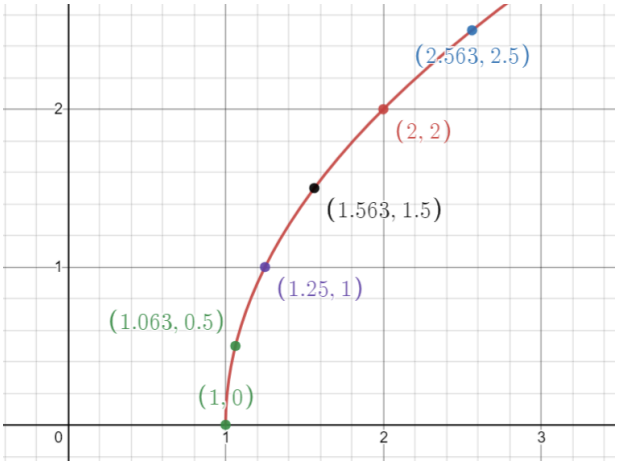
And as you might be able to see, this looks like a bird's-eye-view of the Flamm's paraboloid, but with a twist (no pun intended)!

What the Pattern is Showing

The crochet models consist of two Flamm's paraboloids, one being normal (Schwarzschild), and one having frame-dragging included (Kerr). The simpler normal paraboloid isn't all that useful on its own, but there are very few visual representations of frame-dragging.Together, these could be very useful in showing what frame dragging looks like and how a particle might move around a black hole.

Creating the Pattern:

I graphed the formula for Flamm's paraboloid and found points with equally spaced apart y values (so y=0, 0.5, 1, 1.5 etc) and noted their corresponding x coordinates. That way, I can make each y value be 1 round of crochet, and the x values will tell me how much I need to increase. Basically, when the x coordinate increases, the circumference of the crochet needs to increase by the same amount, which is useful for crochet as we're just making a lot of circumferences stacked on top of each other.



Round	X-value	Stitches needed
1	1	10
2	1.063	11
3	1.25	13
4	1.563	16
5	2	20
6	2.563	26
7	3.25	33
8	4.062	41
9	5	50
10	6.063	61

For example in the graph above, I start with 1 stitch in the 0th row (which is technically the first row), then in the next row that 1 stitch increases to 1.063 stitches. In order to make a tube, I need to start with many stitches, so for example if I cast on 10 stitches, then in the next round I will need 10.63 ~ 11 stitches, and this is how I can increase in accuracy.

The table has the number of stitches set out for a paraboloid that starts with 10 stitches:

The Pattern

I used half double crochet (hdc), but this pattern is customisable.

Black Hole Base

Foundation Round: chain 10 and made a slip stitch into the first chain to join in the round.

Round 1: Chain 2, make 1 hdc in each chain, slip into the first hdc. (10)

Round 2: Chain 2, make 1 hdc in each of next 4 chains, make 2 hdc in next stitch, hdc in each of the rest of the stitches, slip into the first hdc. (11)

Round 3: Chain 2, make 2 hdc in next stitch, hdc in each of next 5 stitches, make 2 hdc in next stitch, hdc in each of the rest of the stitches, slip into the first hdc. (13)

Round 4: Chain 2, * make 2hdc in next stitch, hdc in each of next 3 stitches, * repeat 2 more times, hdc, slip into the first hdc. (16)

From now on make increases evenly spaced throughout the round.

Round 5: Chain 2, hdc in each stitch while making a total of 4 increases, slip into the first hdc. (20)

Round 6: Chain 2, hdc in each stitch while making a total of 6 increases, slip into the first hdc. (26)

Round 7: Chain 2, hdc in each stitch while making a total of 7 increases, slip into the first hdc. (33)

Round 8: Chain 2, hdc in each stitch while making a total of 8 increases, slip into the first hdc. (41)

Round 9: Chain 2, hdc in each stitch while making a total of 9 increases, slip into the first hdc. (50)

Round 10: Chain 2, hdc in each stitch while making a total of 11 increases, slip into the first hdc. (61)

You can keep on going as long as you want, but I stopped here. Below is the finished piece:


Sewing Instructions

The gravitational lines can be backstitched, but you could also use a crochet slip stitch along the top of the fabric. To back stitch, first thread a tapestry needle with a long tail of yarn of a contrasting colour for the best definition.

Make a knot in the back of one stitch in the last row, the end can be woven in later. Poke the needle through the stitch to bring the yarn to the front. To start the back stitch, bring the needle back down the fabric In the stitch from the next row, this creates the first stitch. Then repeat the step but instead bring the needle up from the underside from the next row. Take the needle and bring it back down through the previous row, then bring the needle up from the next row. This forms the repeat of the backstitch. Continue backstitching in either one of the two shapes below.


Use a long piece of yarn so you can work all the lines in one go without working multiple ends. Between each line just simply thread the yarn discreetly through the back of the stitches until you get to where you need to be. Your backstitches will not always be even because some stitches will be worked diagonally, so do not worry! To keep things fairly

even, your backstitches should only be made one per row I.e. each stitch will only be roughly 1hdc stitch long.

Schwarzschild Black Hole

Sew in a straight line from the edge to the centre. Start at the bottom of a stitch and work vertically upwards, adding no horizontal component. Make at least 3 lines to see the pattern best.



Kerr Black hole

Start by stitching a wide spiral by making your stitches very slightly moving diagonally. For my model, each back stitch started from the bottom left of a crochet stitch, and finished at the top middle. After working past the third row, increase the spirality and diagonality of the stitches so that all back stitches go from the bottom left to the top right of each crochet stitch. This should make them look more tightly coiled up when all the lines are complete.




Acknowledgements

The frame-dragging diagram and all information is credited to the amazing thesis written by P.C van der Wijk.